Color difference Delta E - A survey

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Contents

Abstract Color perception is crucial for human existence. In this why, color spaces have been developed to describe mathematically the color that a person can feel with unaided eye. There was a new need to distinguish colors, define them as similar, identical or completely different. However a color-matching technique requires a color palette with perceptually linear characteristics. In this article the most popular colors spaces are presented, as both linear and nonlinear ones due to the perceptual abilities, and are briefly discussed and compared to the sample values.

Keywords: color difference, ΔE, perceptual color spaces,

1. Introduction

The recognition technique for human color vision is in fact an extremely complex process. The color is a psychophysical quantity, acting as an impression during the stimulation of our visual system. The dependence on many external factors and individual human characteristics have a significant influence on the perception and comparison of color experiences. This is even more important if color, besides the shape or size, should be the main parameter of object recognition. With help the mathematical models come, to describe colors that precisely define and describe these experiences by fixed values. Due to the presence of three types of receptors in the human eye, the most common model is three-dimensional. It creates a color solid with independent parameters.

2. The concept of color difference and its tolerance

2.1. Determinants of color perception

The main feature of each image is color[1], and in printing technology (and not only) what is most important is not so much its absolute values, but reproducibility and fidelity to the original or the pattern of colors, ie. Pantone, RAL or HKS.

All printed materials are subject to standardization and determination of their maximum acceptable limit differences. Also, a very important process during image reproduction is an I/O device calibration, and the effectiveness of this operation is described by term "color difference".

Determine the appropriate color spaces require changes in perception of these spaces and their trichromatic components X, Y, Z stimulating human visual receptors, which was made by the International Commission on Illumination *(CIE, Commission Internationale de l'Eclairage)*, calculating the reference stimulus, ie.: L*,u*,v* or L*,a*,b*. The idea was to create a linear color space in which **the distance between the points defining individual colors would be proportional to the perceptual difference between them** (perceptual color spaces) and to present colors with the coordinates describing one of their key attributes, ie: lightness, chroma (saturation) and hue (ie. L, C, h), [28].

The interest and demand for the calculation of color difference increased with the introduction of **a standard observer**[2] to colorimetry in 1931 by the CIE , [35], because it allowed for accurate determination of the tolerance of prints, colors, materials, inks, images, and multimedia equipment calibration.

[1]The definition of *color* will be understood as any visual impression caused by the light. And the term *chroma* will be used to differentiate color experiences (with shades) from the colourless experiences (gray), and to deepen the differences in color characteristics in visual experience.

[2]CIE has defined a colorimetric **Standard observer** , to allow for describing the color via numbers. It represents the average person with normal color perception. The CIE has defined two standard observers with different observation angles: $2°$ and $10°$ An observer with the angle of $2°$ corresponds to the observation of an object with a small size (using an optical instrument), while an observer with the angle of $10°$ corresponds to the observation of an object in normal conditions.

2.2. Difference in color and tolerance for color of product

From our own experience we know that color perception is affected by many different factors, including the following:

- physical properties of the observed object, especially its absorption characteristics,
- spectral composition of the light source and characteristics of the environment that it passes through,
- observer's visual system properties, and the state of his/her neural centers and transmission receptors,
- proximity of other objects, their properties, and experience acquired from observation of similar objects.

The observed difference in the color (perceptual difference) is a psychophysical difference noticeable by the observer, determined by the actual observation of two samples.

The calculated difference in the color depends on the color model. Because the color stimulus can be represented as a point in space, the difference in color ΔE between two stimuli is calculated as the distance between the points representing these stimuli.

Tolerance for the product is a range of values, within which the product is considered acceptable. Any product that falls outside this range is not acceptable. Tolerance for the color of a product can be determined either visually or by using an instrument with a scale of colors available for this instrument. To determine the tolerance a perfect, or nearly perfect product standard is required, as well as products to be identified as either acceptable or not. Determining the level of tolerance, ie. 1 ΔE_{FMCII} units defines what is acceptable and what is discarded. However, the most important thing is to understand the **basis of difference detection.**

There are two levels of visual differences in color used to establish the tolerance:

- minimal perceptual difference, which is defined as a visually *just noticeable difference* between the pattern and sample,
- maximal perceptual difference, which is defined as *the highest acceptable difference between the pattern and the sample.* This type (level) of difference in color is essential for determining the color; any higher difference causes rejection of the sample.

After determining the tolerance values, the following general rules are also valid:

1. people perceive, as the most improper, the difference in shades,
2. people often tolerate slightly larger differences in chroma than in shade,
3. people tolerate differences in brightness more easily than differences in chroma or in shade.

Establishment of tolerance. Tolerance is based on the estimation measurement of acceptable and unacceptable samples, and an ideal standard product for each color of the product. Also special needs of tolerance are taken into account. The tolerance is smaller for dark colors, and greater for bright colors.

It is not appropriate to use ΔE as the tolerance, because it ignores the difference spreading to all dimensions of space (e.g., L, a i b). When the difference is concentrated in one dimension, this may be unacceptable. If the tolerance amounts to 1 ΔE unit, this difference should be 0.57 for each of the dimensions: L, a and b, which would probably be acceptable visually ($\Delta E = \sqrt{0.57^2 + 0.57^2 + 0.57^2}$=1). However, when the sample is ideal in L and in b, but not in a (although still within the tolerance limits), the sample is no longer acceptable.

3. An early period in ΔE formalization

3.1. JND units and the ΔE_{DN} formula

The first known formula for the ΔE came from Dorothy Nickerson in 1936 (*Just Noticeable Difference, JND*), [11]. It used Munsell color system [25] and was given by:

$$\Delta E_{DN} = \frac{2}{5} C \Delta H + 6 \Delta V + 3 \Delta C.$$

3.2. Judd NBS units, Judd ΔE_J and Judd-Hunter ΔE_{NBS} formulas

Judd formula for ΔE for Judd UCS space[3] was [3]:

$$\Delta E_J = \sqrt{\Delta L^2 + \Delta C^2},$$

where L is the Lightness, C is the Chroma.

Later Judd defined the size of the color difference unit as *such a value that smaller differences may be omitted in ordinary commercial transactions with the required color comparisons, but greater - not.* This became the basis for determining the so-called *NBS color difference units* (from *National Bureau of Standards U.S.*) - averaged maximum acceptable difference in the series of measurements of dye (DB Judd, 1939). These were defined in the space of the triangular diagram of UCS (1935 r., [3]).

Judd formula has been modified by R.S. Hunter (1942) based on the Cartesian coordinates *alpha-beta* chromacity diagram, to the form (Judd-Hunter formula (NBS), []):

$$\Delta E_{NBS} = f_g \sqrt{[221 \overline{Y}^{1/4} \sqrt{(\Delta\alpha)^2 + (\Delta\beta)^2}]^2 + [k(\Delta Y^{1/2})]^2},$$

where: $\overline{Y} = \frac{Y_1 + Y_2}{2}$, $\Delta Y^{1/2} = Y_1^{1/2} - Y_2^{1/2}$

$$\alpha = \frac{2.4266x - 1.3631y - 0.3214}{1.0000x + 2.2633y + 1.1054}, \beta = \frac{0.5710x + 1.2447y - 0.5708}{1.0000x + 2.2633y + 1.1054},$$

f_g - gloss factor,

k - proximity factor of compared samples.

No other formula for ΔE takes such factors in account, although the impact of gloss and other samples in the vicinity ΔE is generally noticeable.

The ΔE_{NBS} results computed for the bright colors are close to the results calculated according to the latest rules for the HunterLab and CIELAB color space. For dark colors, these differences are much more noticeable.

3.3. Adams chromatic valence color space and the ΔE_A formula

Adams color space E.Q. (1942 r.) is a new class of space derived from Adams theory of color vision, characterized by a model taking into account the view of opposing processes, and the experimentally

[3] An important application of this coordinate system is its use in finding, in any series of colors, the one most resembling a neighboring color of the same brilliance; for example, finding of the nearest color temperature for a neighboring non-Planckian stimulus. The method is to draw the shortest line from the point representing the non-Planckian stimulus to the Planckian locus.

determined ratio of 5 / 2 in the chromatic components red/green vs. blue/yellow color. Adams called this class of space "valued tonally". Have almost the same radial distance for the same changes in Munsell color. The best known evaluated tonally spaces are the CIELUV and HunterLab spaces (and their successors). Adams called this class of spaces "chromatically valued". They have almost the same radial distance for the same changes in Munsell color. The best known evaluated chromatically spaces are the CIELUV and HunterLab spaces (and their successors).

A color space with chroma estimation has three components, [30]:

- V_Y - Munsell-Sloan-Godlove value function[4]: $V_Y^2 = 1.4742Y - 0.004743Y^2$;
- $V_X - V_Y$ - chromatic direction red/green, where V_X is used for $(y_n/x_n)X$ instead of Y;
- $V_Z - V_Y$ - chromatic direction blue/yellow, where V_Z is a function used for $(y_n - z_n)Z$ instead of Y.

The diagram is a plot of chromatic values $V_X - V_Y$ (horizontal axis) and $0.4(V_Z - V_Y)$ (vertical axis). The 0.4 factor correlates the radial distance from the white point of the Munsell color along any hue radius (i.e., to make the diagram perceptually uniform). For the gray plane it is: $(y_n/x_n)X = Y = (y_n/z_n)Z$, and hence $V_X - V_Y = 0,\ V_Z - V_Y = 0$. In other words, the white point is at the origin.

Chromaticity. The chromatic brightness scales were removed, leaving two dimensions. Two lamps with the same spectral power, but different brightness, will have the same chromatic coordinates. Chromatic diagram CIE (x, y) is perceptually very uneven; small perceptual changes, e.g. in chromatic green corresponds to large distances, while large perceptual changes in other chromatic shades are usually much smaller. Adams [4] suggested a relatively simple uniform chromaticity scale:

$$\frac{y_n}{x_n}X - Y \text{ and } \frac{y_n}{z_n}Z - Y,$$

where x_n, y_n and z_n are white colors of the reference object (suggested in normalization).

Objects that have the same chromacity coordinates as a white object usually appear as neutral or nearly such, and standardization in this state makes sure that their coordinates are at the beginning. Adams outlined the horizontal axis, and then multiplied by 0.4 the vertical axis. The scaling factor is to make sure that the curve of a constant color (saturation) lies on a circle. Distances from the centre along any radius are proportional to the colorimetric purity.

The chromacity diagram changes with brightness, so Adams normalized each element by the value of tristimulus Y:

$$\frac{y_n X}{x_n Y} = \frac{x/x_n}{y/y_n} \text{ and } \frac{y_n Z}{z_n Y} = \frac{z/z_n}{y/y_n}.$$

He noticed that these expressions only depend on chromatic samples, and called them *constant brightness chromacity diagrams*. This chart does not have the white point at the beginning, but at the point(1,1).

Chromatic valence. Chromatic valence spaces include two of perceptually uniform components: the brightness scale and the chromacity scale. The brightness scale is defined using Newhall-Nickerson-Judd[5] function describes the following axis of color space:

$$Y = 1.2219V_J - 0.23111V_J^2 + 0.23951V_J^3 - 0.021009V_J^4 + 0.0008404V_J^5.$$

[4]Munsell-Sloan-Godlove function is a function binding *relative illumination* with Munsell*value*. Munsell, Sloan and Godlove suggested a 2-degree parabola for the neutral scale values in the Munsell color system. ([1, 2]):

$$V^2 = 1.4742Y - 0.004743Y^2.$$

[5]The Newhall-Nickerson-Judd function is a function binding the Munsell *value* and the *reflectivity*.

The remaining axes are given by Y:

$$\frac{X/x_n}{Y/y_n} - 1 = \frac{X/x_n - Y/y_n}{Y/y_n} \quad (1)$$

$$\frac{Z/z_n}{Y/y_n} - 1 = \frac{Z/z_n - Y/y_n}{Y/y_n}. \quad (2)$$

It is important what he used in his Hunter Lab space. Like with the chromacity value, these functions are plotted with the factor of 21/8, to achieve nearly equality to the radial distance for equal changes in the Munsell color.

Color difference. Adams' color spaces are based on the Munsell values for lightness. Defining chromatic valence components:

$$W_X = \left(\frac{x/x_n}{y/y_n} - 1\right) V_J \text{ and } W_Z = \left(\frac{z/z_n}{y/y_n} - 1\right) V_J,$$

we can determine the difference between two colors as:

$$\Delta E_A = \sqrt{(0.5\Delta V_J)^2 + (\Delta W_X)^2 + (0.4\Delta W_Z)^2},$$

where V_J is a Newhall-Nickerson-Judd function, and the 0.4 factor is used to make the differences in W_X and W_Z perceptually equivalent to each other.

In chromaticity valued color spaces, they are given by $W_X = V_X - V_Y$ and $W_Z = V_Z - V_Y$. This means that the difference is:

$$\Delta E_A = \sqrt{(0.23\Delta V_Y)^2 + (\Delta W_X)^2 + (0.4\Delta W_Z)^2},$$

where Munsell-Sloan-Godlove value function is used.

3.4. MacAdam ellipses and the ΔE_{FMCII} formula

In 1942 r. MacAdam introduced the concept of *MacAdam ellipses*, covering *perceptual field inhomogeneity*. Further studies conducted among others by MacAdam, Brown, Chickering and Friele led to the expansion of MacAdams' *ellipsoidal areas* theory and creation of (based on test results for *just noticeable difference*) two other color difference formulas: ΔE_{FMCI} and ΔE_{FMCII}, and the creation of the CIELuv diagram. The *FMC II* (*F - Friele, M - MacAdam, C - Chickering*) formula was based on a linear transformation of *XYZ* to the *P*, *Q*, *S* coordinates given by:

$$\begin{aligned} P &= 0.724X + 0.382Y + 0.098Z \\ Q &= -0.48X + 1.37Y + 0.1276Z \\ S &= 0.686Z \end{aligned}$$

Here *X*, *Y*, *Z* are the tristimulus components. The formula for color difference is:

$$\Delta E_{FMCII} = \sqrt{(\Delta C)^2 + (\Delta L)^2}$$

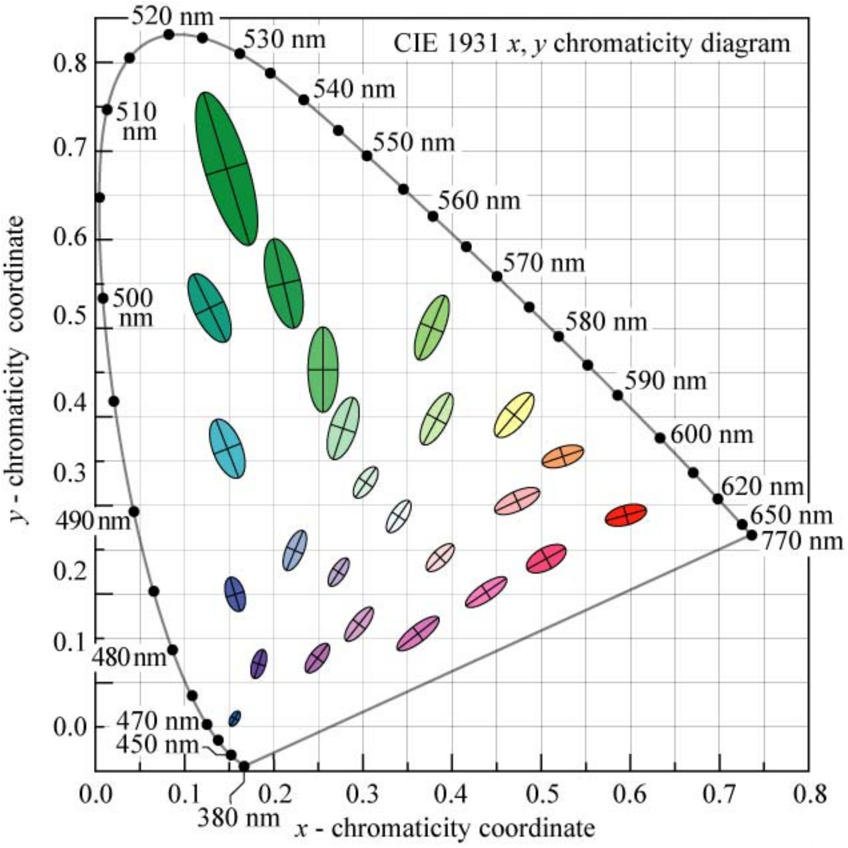

Figure 1: MacAdam ellipses plotted in the CIE 1931 (x,y) chromacity diagram [32].

where: ΔL and ΔC describe the brightness difference and the chromacity difference (ΔC_{r-g} red-green and ΔC_{y-b} yellow-blue) between the assessed colors;

$$\begin{aligned}
\Delta C &= K_1 \Delta C_1;\ \ \Delta C_1 = \sqrt{\left(\frac{\Delta C_{r-g}}{a}\right)^2 + \left(\frac{\Delta C_{y-b}}{b}\right)^2} \\
\Delta C_{r-g} &= \sqrt{\frac{Q\Delta P - P\Delta Q}{P^2+Q^2}};\ \ \Delta C_{y-b} = \sqrt{\frac{S\Delta L_1}{P^2+Q^2}} - \Delta S \\
\Delta L &= K_2 \Delta L_2;\ \ \Delta L_2 = \frac{0.279\Delta L_1}{a};\ \ \Delta L_1 = \sqrt{\frac{P\Delta P + Q\Delta Q}{P^2+Q^2}} \\
K_1 &= 0.55669 + 0.04934Y - 0.82575 \cdot 10^{-3}Y^2 + \\
&\quad 0.79172 \cdot 10^{-5}Y^3 - 0.30087 \cdot 10^{-7}Y^4 \\
K_2 &= 0.17548 + 0.027556Y - 0.57262 \cdot 10^{-3}Y^2 + \\
&\quad 0.63893 \cdot 10^{-5}Y^3 - 0.26731 \cdot 10^{-7}Y^4 \\
a &= 17.3 \cdot 10^{-5}(P^2+Q^2)/[1 + 2.73P^2Q^2/(P^4+Q^4)] \\
b &= 3.098 \cdot 10^{-4}(S^2 + 0.2015Y^2)
\end{aligned}$$

ΔP, ΔQ, ΔS are differences in P, Q and S values of assesed colors; K_1 i K_2 are parameters. MacAdam and Simon simplified the expressions for K_1 and K_2 to the form:

$$K_1 = 0.054 + 0.46Y1/3, \quad K_2 = 0.465K_1 - 0.062.$$

The ΔE_{FMCII} formula was designed so that by adopting the value of 1 it represents *just noticeable difference* in color. It maps well the MacAdam ellipses containing colors indistinguishable to the eye when the luminance Y=10.69=const. The K_1 parameter is used to simulate the increase / decrease in ellipses as a function of the luminance factor Y, and to establish a fixed point of saturation in the Munsell system. K_2 parameter allows for conversion of Friele differences in brightness to Simon-Goodwin type of difference in brightness.

The ΔE_{FMCII} formula is still an option for calculating the color difference in modern spectrophotometers and colorimeters.

4. The $ANLab$ model and ΔE formulas

4.1. The $ANLab$ model

The $ANLab$ color space was created as a result of E.Q. Adams' and D. Nickerson's research. In 1942, Adams created a scale of chromacity (chroma) values , which took into account Hering's theory on opponent colors[6]. He reduced X and Z to Y, creating a opposing color plane. While adding the coefficients of shrinking CIEXYZ coordinates, he obtained the coordinates V_X, V_Y and V_Z. In this way, the space known as **ANLAB-40** was created (1942) (A - for Adams, N - for Nickerson, LAB - for the three axes, 40 - the coefficient of $V_X - V_Y$).

The ANLAB system defines the coordinates L, a and b as follows:

$$L = 9.2V_Y, \quad a = 40(V_X - V_Y), \quad b = 16(V_Y - V_Z).$$

4.2. The ΔE_{AN} formula

The formula for calculating the color difference was given the form:

$$\Delta E_{AN} = 40\sqrt{0.23\Delta V_Y{}^2 + \Delta(V_X - V_Y)^2 + 0.4\Delta(V_Z - V_Y)^2},$$

where: ΔV_Y is the difference in brightness coordinates $\Delta(V_X - V_Y)$, and $\Delta(V_Z - V_Y)$ are the differences in color coordinates.

In 1971, ΔE_{AN} became an ISO standard for the textile industry, [26]. In 1973, the CIE recommended ANLAB system to calculate the color difference, and formed a subcommittee to transform the ANLAB system into CIELAB system.

4.3. McLaren ΔE_{McL} and McDonald ΔE_{JPC79} formulas

Based on the $ANLab$ space McLaren (1976) - a pioneer of adding weighting factors into the formulas for ΔE, simplified the formula previously published by McDonald [8], and introduced a weight factor doubling the effect of hue compared to chromacity and brightness:

$$\Delta E_{McL} = \frac{\sqrt{(\Delta L)^2 + (\Delta C)^2 + (2\Delta H)^2}}{1 + 0.02C}.$$

[6] In 1878 Hering E. announced the *opponent colors* theory, according to which there are ganglion cells in the eye sensitive to radiation (received by three types of cones) from three pairs of opposing colors: red and green, yellow and blue, black and white. In each cell, impulses cause the formation of mixed colors (already partly in the eye). This theory is the basis of the color spaces recommended since 1976 by the CIE (CIELAB, CIELCh, CIELUV)

McDonald, who was also an advocate of introducing weighting factors in the formulas for ΔE, brought in his own formula for ΔE with weights, known as the **JPC79 formula**, of the form:

$$\Delta E_{JPC79} = \sqrt{\left(\frac{\Delta L}{S_L}\right)^2 + \left(\frac{\Delta C}{S_C}\right)^2 + \left(\frac{\Delta H}{S_H}\right)^2},$$

where S_L depends on L, S_C depends on C, S_H depends on H. L, C i H are computed from the $ANLab$ space.

4.4. The Hunter color system and the ΔE_H formula

In 1948, R. S. Hunter proposed a color space more uniform in the perception, and the formula for ΔE and the values readable directly from a photoelectric colorimeter. The formula evolved in the 1950s and 1960s, assuming the current shape in 1966. CIEXYZ values were transformed into Hunter L, a, b coordinates with the arrangements:

- L coordinate for representing brightness following in $0 \leq L \leq 100$, L=0 for black, L=100 for a perfectly reflecting diffuser;
- A coordinate for representing colors on the red-green axis, with positive values for red, and negative for green;
- B coordinate for representing colors on the blue-yellow color axis, with positive values for yellow, and negative for blue.

The values of L, A and B coordinates are derived via the following formulas for the standard illuminant C:

$$L = 10Y^{1/2}, \ a = [17.5(1.02X - Y)]/Y^{1/2}, \ b = [7.0(Y - .847Z)]/Y^{1/2},$$

while the color difference is calculated from the formula:

$$\Delta E_H = \sqrt{(\Delta L)^2 + (\Delta a)^2 + (\Delta b)^2},$$

where: ΔL is the difference in brightness between two vivid surfaces, and Δa and Δb are the differences in the color coordinates A and B, respectively.

5. ΔE formulas in uniform color spaces

5.1. Uniform space CIE 1960 *USC*

The abbreviation *USC* has been developed in the following different versions: *Uniform Color Space*, *Uniform Chromaticity Scale*, and *Uniform Chromaticity Space*. In this space one does not define the components of light or brightness, but the Y coordinate of the tristimulus XYZ color space, or the brightness ratio.

The idea came from Judd, who discovered that a more uniform color space can be obtained by projection of the tristimulus $CIEXYZ$ in the form of [3]:

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix} = \begin{pmatrix} 3.1956 & 2.4478 & -0.1434 \\ -2.5455 & 7.0492 & 0.9963 \\ 0.0000 & 0.0000 & 1.0000 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

Judd was the first to use this type of transformation, and many imitate him in this. Transforming the RGB space into the chromacity space (called UCS Judd space), he received the following formulas for the coordinates u and v:

$$u = \frac{0.4461x + 0.1593y}{y - 0.15735x + 0.2424}, \quad v = \frac{0.6581y}{y - 0.15735x + 0.2424},$$

which MacAdam simplified for computing purposes to the form:

$$u = \frac{4x}{12y - 2x + 3}, \quad v = \frac{6y}{12y - 2x + 3}.$$

UCS Judd feature space is such that isotherms are perpendicular to Planckian locus [7]. So for now it is used for calculating the color correlated temperature.

Its relation to the $CIEXYZ$ space is as follows:

$$U = \frac{2}{3}X, \quad V = Y, \quad W = \frac{1}{2}(-X + 3Y + Z), \quad Z = \frac{1}{2}(3U - 3V + 2W)$$
$$u = \frac{4X}{X + 15Y + 3Z}, \quad v = \frac{6Y}{X + 15Y + 3Z},$$

and to the $CIELUV$ space- as given below:

$$u' = u, \quad v' = \frac{3}{2}v.$$

The $CIE\ UCS$ space was - thanks to the efforts of MacAdam - recommended by the CIE (1960) for use in perceptual situations when there is need for greater uniformity than that offered by the color space (x, y), and then adopted as the UCS standard. Hence the other name of this space - MacAdam space (u, v). And as a space of uniform chromaticity it was converted into CIE 1976 UCS space ($CIELAB$).

5.2. OSA color space and the ΔE_{OSA} formula

In 1960 **OSA** (*Optical Society of America*) created the first model of a perceptually uniform color space [34]. The three coordinates which describe the color are:

- L - lightness; interval -10<L<8; L=0 reflects 30% of neutral gray;
- j - color yellowness; interval -12<j<11; positive j values for the yellow color, negative - for the blues;
- g - color greeness; interval -14<g<10; positive g values for green, negative - for red.

The ΔE formula determined by the OSA system is based on the Euclidean distance in color space Ljg:

$$\Delta E_{OSA} = \sqrt{2(\Delta L)^2 + (\Delta g)^2 + (\Delta j)^2}$$

and is not suitable for the measurement of *just noticeable difference* of colors.

[7] The science of Planckian locus (location) is the path or the place where the color of a black body radiating light is appropriate to the black body temperature changes in a specific color space. It goes from deep red at low temperatures through orange, yellowish white, white, and finally bluish white at very high temperatures

5.3. Chromacity space CIE 1964 and the ΔE^*_{CIE} formula

In 1964, the CIE recommended the use of an auxiliary reference stimuli $U*$, $V*$, $W*$, resulting in the shift of the X, Y, Z. CIE 1964 color space (U^*, V^*, W^*), also denoted as $CIEUVW$, was derived by Wyszecki from CIE 1960 UCS space using the formula:

$$U^* = 13W^*(u - u_0),\ V^* = 13W^*(v - v_0),\ W^* = 25Y^{1/3} - 17,$$

where (u_0, v_0) is the white point, and Y is the brightness of the object tristimulus. Stars in superscripts describe a perceptual uniformity of the color space greater than in the previous one.

Wyszecki's intention was to calculate the color difference without maintaining a constant brightness. He defined the brightness component W^* by simplifying the expressions suggested earlier in [6, 7]. The chromacity components U^* i V^* were defined so that the white point was transformed into the origin of the space (as in case of Adams with the color valence), which is useful, because it expresses the position of chromacity with constant saturation as $(U^*)^2$+ $(V^*)^2$ = C= const. Moreover, the chromacity axes are scaled by brightness, making it easier to increase / decrease saturation when the brightness ratio increases / decreases, and the chromacity (u, v) is kept constant.

The chromacity coefficients were chosen based on a division of the Munsell system. It was assumed that the difference in brightness ΔW=1 corresponds to the difference in chromacity $\sqrt{\Delta U^2 + \Delta Y^2}$=13. With the coefficients selected in this way, the color difference in the $CIEUVW$ (CIE units) is given by the Euclidean distance:

$$\Delta E_{CIEUVW} = \sqrt{(\Delta U^*)2 + (\Delta V^*)^2 + (\Delta W^*)^2}.$$

In practice it turned out that such a color difference ΔE^*_{CIE} deviated significantly from the visual sensations of the observer, especially for dark surfaces. It was necessary to use two new, more perceptual color spaces resulting from the transposition of X, Y, Z to L, u, v and L, a, b, denoted in short as $CIELUV$ and $CIELAB$ spaces.

5.4. The ΔE_{Luv} formula in the $CIEL^*u^*v^*$ space

The $CIEL^*u^*v^*$ space (or $CIELUV$), is one of the Adams space with valence of colors and is complemented by a space CIE 1964 (U^*, V^*, W^*, CIEUVW). Its difference is a small brightness scale modification and uniform chromacity modification scale by a factor of 1.5 on the axis v' (v'=1.5 v) compared to its predecessor from 1960. In this space, Judd transformation is used to adapt to the white point (translational, unlike the CIELAB space with the von Kries transformation[8]). It is easily derived from the CIE XYZ space with aim of perceptual uniformity, and is applied in computer graphics that operates on chromatic lights. There are limitations however, on the additive mixing of lights: the mixtures should have a constant brightness. The (nonlinear) dependencies on L^*, u^* and v^* are as follows [30]:

$$\begin{aligned} L^* &= \begin{cases} \left(\frac{29}{3}\right)^3 Y/Y_n & Y/Y_n \leq \left(\frac{6}{29}\right)^3 \\ 116\,(Y/Y_n)^{1/3} - 16 & Y/Y_n > \left(\frac{6}{29}\right)^3 \end{cases} \\ u^* &= 13L^*(u' - u'_n) \\ v^* &= 13L^*(v' - v'_n). \end{aligned}$$

[8]Von Kries transformation is one of the color modeling adaptation methods. The method consist in applying a gain to each of the human cone cell spectral sensitivity responses, so as to keep the adapted appearance of the reference white constant

The u'_n i v'_n values are the coordinates of the white point, and Y_n - its luminance. CIELUV with a cylindrical coordinate system is known as $CIELCh_{uv}$, where C^*_{uv} is its chroma and h_{uv} - hue:

$$C^*_{uv} = \sqrt{(u^*)^2 + (v^*)^2}, \quad h_{uv} = \arctan \frac{v^*}{u^*}$$

. The correlated saturation is defined as:

$$s_{uv} = \frac{C^*}{L^*} = 13\sqrt{(u' - u'_n)^2 + (v' - v'_n)^2}.$$

The formula for the difference in color is the Euclidean distance (L^*, u^*, v^*):

$$\Delta E^*_{uv} = \sqrt{(\Delta L^*)^2 + (\Delta u^*)^2 + (\Delta v^*)^2}$$

It shows that the distance of color $\sqrt{(\Delta u')^2 + (\Delta v')^2} = 13$ corresponds to the same ΔE^*_{uv} as the difference in brightness ΔL = 1, which is similar to the CIEUVW space. For calculating the distance between the colors, CIELCH can also be used with a difference in hue in the form $\Delta H^* = \sqrt{C^*_1 C^*_2} 2 \sin(\Delta h/2)$, where $\Delta h = h_2 - h_1$.

6. The $CIEL^*a^*b^*$ color space and ΔE_{Lab} formulas

6.1. Chromacity system $CIEL^*a^*b^*$

In the $CIEL^*a^*b^*$ space, every color is described by three components: L^* - lightness, where 0 means black, and 100 is the maximum light intensity which is still visible without causing eye damage; a^* – color in the green÷ red field (-128,+127), b^* – color in the blue÷ yellow field (-128, +127). In the middle $(a^* = 0, b^* = 0)$ only gray values exist. In this space, all the colors visible and distinguishable for human eyes can be represented. Moreover, the visible spectrum of the colors defined in this way is only 40 % of all the colors of this space [9].

The $CIEL^*a^*b^*$ space is based on the so-called **opponent color model**: colors lying opposite each other on both sides of the plane (along L^*) and on the a^*b^* plane can not be seen simultaneously. This means that either *dark or bright* is seen , either *red or green* and either *yellow or blue*:

- L^* = *black/white*,
- $+a^*/-a^*$= *red/green*,
- $+b^*/-b^*$ = *blue/yellow*.

The $CIEL^*a^*b^*$ space is a mathematical transformation of the *CIEXYZ* space, defined as follows, [33]:

$$\begin{aligned}
L^* &= 116(Y/Y_0)^{1/3} - 16 \\
a^* &= 500((X/X_0)^{1/3} - (Y/Y_0)^{1/3}) \\
b^* &= 200((Y/Y_0)^{1/3} - (Z/Z_0)^{1/3})
\end{aligned}$$

[9] $CIEL^*a^*b^*$ is similar to $CIEL^*C^*h^*$ (L^* - lightness, C^* - chroma(saturation), h^* - hue). The difference between them is the different coordinate systems used to describe the two spaces: the $CIEL^*a^*b^*$ space is described in Cartesian coordinates, while the $CIEL*C*h*$ space - in cylindrical coordinates (where each of them describes one of its key attributes: brightness, saturation and hue; this makes it easier to relate the values to earlier systems based on physical samples, such as the Munsell system). The relationships between their respective coordinates are therefore as follows: $L^* \equiv L^*$, $C^* = \sqrt{a*^2 + b*^2}$, $h^* = arctan(b^*/a^*)$.

(where: $X_0 = 94.81,\ Y_0 = 100.0,\ Z_0 = 107.3$ are the coordinates of nominally white body color for the CIE D65 illuminant (Y_0 set to 100)), introduced to obtain certain research data on human perception of the difference between colors. The coefficients in these equations were determined by thousands observations, in order to reproduce as closely as possible, the mathematical model of color difference perception by the human eye.

It is assumed that the $CIE - L^*a^*b^*$ space is to be perceptually uniform, meaning that the colors that are at the same distance ΔE from each other should be seen as equally different from each other [10].

6.2. The first formula on ΔE_{Lab}

In case of the $L^*a^*b^*$ space, the ΔE_{Lab} difference between two colors is calculated by the formula:

$$\Delta E_{Lab} = \sqrt{(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2}$$

according to the formula for Euclidean distance between two points in the $CIE - L^*a^*b^*$ space. $\Delta E*_{ab}$ calculated using the formula equals 2.3 JND (Just Noticeable Difference) units coming from D. Nickerson:

$$\Delta E^*_{ab} \approx 2.3 JND.$$

Uniformity (conformity of the calculated and perceived color difference) achieved by its use is significantly better than for the XYZspace, and much better than its predecessors: CIE 1960 UCS and OSA. However, it is not optimal: the perceptual color difference can not be uniquely determined by ΔE_{Lab} either.

A standard observer sees the difference in color as follows (Fig. 2). When:

- $0 < \Delta E < 1$ - observer does not notice the difference,
- $1 < \Delta E < 2$ - only experienced observer can notice the difference
- $2 < \Delta E < 3.5$ - unexperienced observer also notices the difference,
- $3.5 < \Delta E < 5$ - clear difference in color is noticed,
- $5 < \Delta E$ - observer notices two different colors.

These data represent experimentally verified statistics.

6.3. $\Delta E_{CMC(l:c)}$ formulas

Manipulating properly the brightness and saturation coefficients, British Standard 6923 proposed (1994)the way to express color difference under the name of CMC (*l:c*) (developed in 1984 by Clark and others [10, 15]). The equation for calculation of ΔE_{CMC} has the form:

- $\Delta E_{CMC} = \sqrt{\left[(\frac{\Delta L^*}{lS_L})^2 + (\frac{\Delta C^*_{ab}}{cS_C})^2 + (\frac{\Delta H^*_{ab}}{S_H})^2\right]},$

where:

[10] In the color science, an **absolute color space** exist, which is understood in two ways; a) A color space in which the perceptual difference between colors is directly related to distances between colors as represented by points in the color space, or b) A color space in which colors are unambiguous, that is, where the interpretations of colors in the space are colorimetrically defined without reference to external factors.

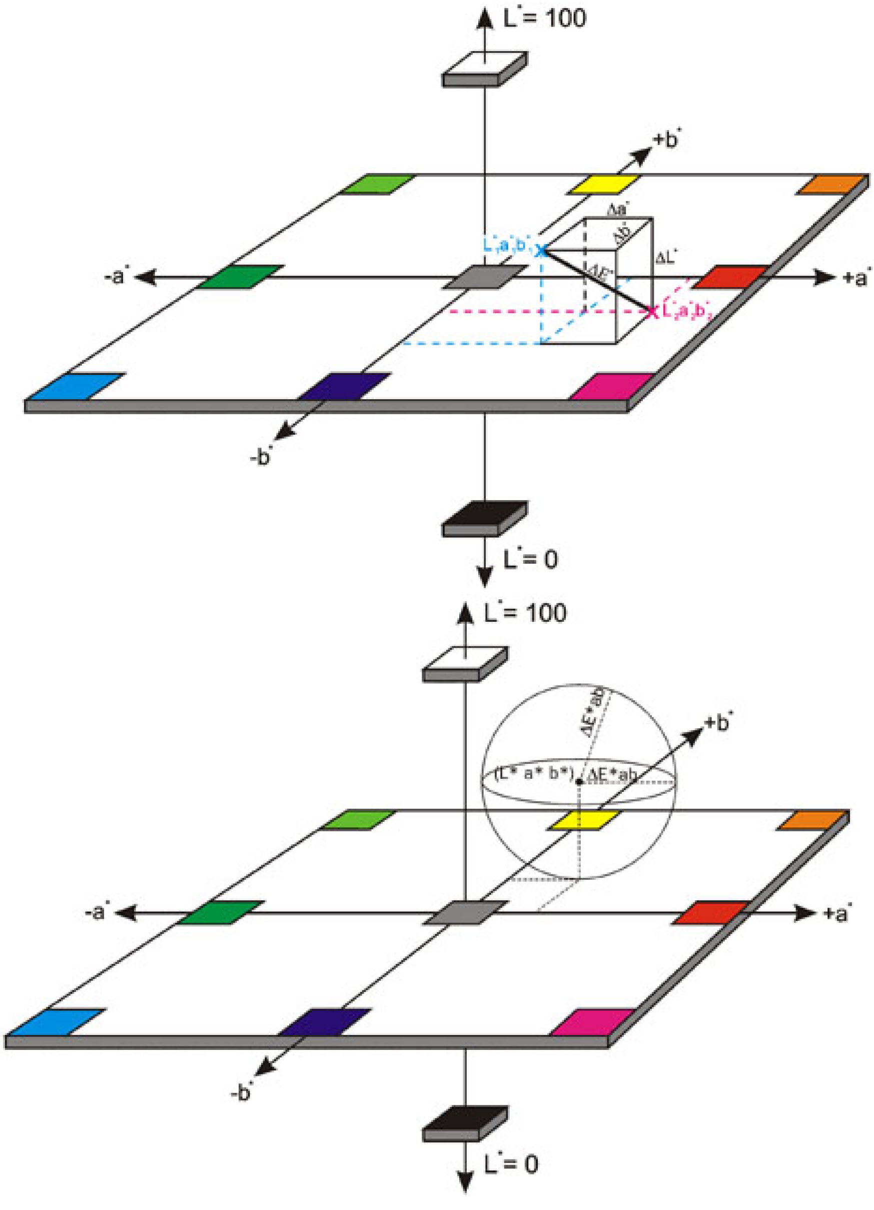

Figure 2: Graphical interpretation of the color difference and color space different from the pattern of a value less than ΔE^*_{ab}, [33]

- ΔL^*, ΔC^*, ΔH^* - differences of two color parameters being compared,
- ***l*** and ***c*** - brightness and saturation respectively,
- S_L, S_C, S_H - additional functions described on the formulas :
- $S_L = \frac{0.040975L^*}{1+0.01765L^*}$ for $L^* \geq 16$,
- $S_L = 0.511$ for $L^* < 16$
- $S_C = \left[\frac{0.0638C*_{ab}}{(1+0.0131 \times C*_{ab}}\right] + 0.638$
- $S_H = S_C(FT + 1 - F)$,

where:

- $F = \sqrt{\frac{(C*_{ab})^4}{[(C*_{ab})^4+1900]}}$,
- $T = 0.56 + abs\,[0.2cos(h + 168)]$,
- for $164^0 \leq h \leq 345^0$.

otherwise:

- $T = 0, 36 + abs\,[0, 4cos(h + 35)]$,
- $h = arctan(\frac{b^*}{a^*})$ is the hue angle for each color, in the range: $0^0 \leq h \leq 360^0$.

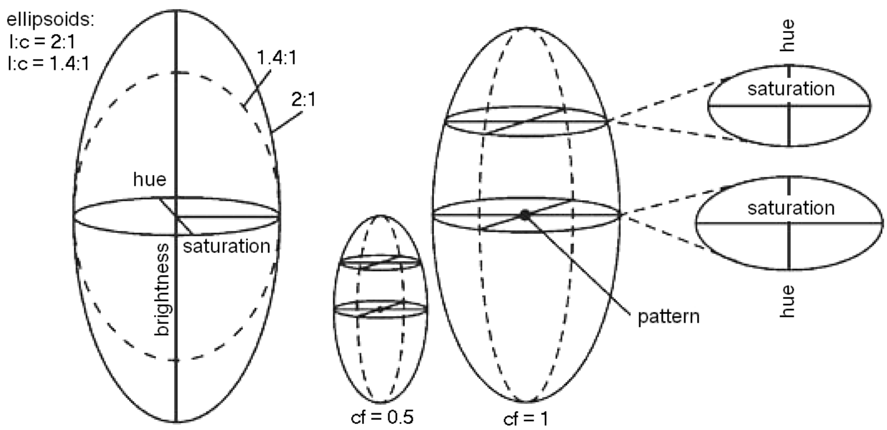

Figure 3: Ellipsoids defining the unnoticeable color area difference and tolerance unit in the CMC(*l:c*) formula [28]

The introduction of additional features allowed for defining a more accurate color difference. The geometrical representation of the space ΔE_{CMC} is an ellipsoid, where the size and orientation are different depending on the location in space. When *l=2* i *c=1*, the equation sets the ratio of the three factors ($S_L : S_C : S_H$) to correlate with visual assessment of the observer. Most of the perceived changes in the CMC space, expressed as numerical values, are changes in hue and saturation - changes in the brightness parameter are insignificant. Assuming a particular value for tolerance of ΔE_{CMC}, the colors acceptable for a given value can be placed inside an ellipsoid, whose center is the value of the pattern. On its basis the ***coefficient cf*** was obtained, which is tolerance limit for each sample's evaluated color; lS_L, cS_C, S_H are the parameters describing the size of the ellipse. The formula $\Delta E_{CMC} \leq cf$ defines the tolerance range, [17].

6.4. The $\Delta E_{BFD(l:c)}$ formula

In 1987 Luo and Rigg [12, 13] developed the BFD formula for the difference in color, in which a correction formula CMC (l: c) was introduced in the blues, giving it the form:

$$\Delta E_{BFD(l:c)}=\sqrt{\left(\frac{\Delta L^*_{BFD}}{l}\right)^2+\left(\frac{\Delta C^*}{cD_c}\right)^2+\left(\frac{\Delta H^*}{D_H}\right)^2+R_T\left(\frac{\Delta C^*\Delta H^*}{D_C D_H}\right)},$$

where $L_{BFD} = 54.6log_{10}(Y + 1.5) - 9.6$,$D_c$ depends on the arithmetic average of the color values for the compared colors 1 and 2, D_H depends on the arithmetic means (or means) of colors and shades of colors being compared, and R_T is the values of correlation between the factors: it depends on the arithmetic mean of the compared chroma colors, and on the arithmetic average value of the color shades being compared. The chroma values and shades are calculated in the CIELAB space.

6.5. The ΔE^*_{94} formula

In 1995, the CIE committee introduced a formula for industrial applications with small differences in the ΔE_{CMC} color difference, called CIE94 [19]. The total ΔE^*_{94} difference in colors of the samples was the weighted Euclidean distance in the $L^*a^*b^*$ space with rectangular coordinates ΔL^*, ΔC^*_{ab} and ΔH^*_{ab}. The amplitude of the perceived ΔV color difference with the full color difference is described by the coefficientK_E:

$$\Delta E^*_{94} = K_E \Delta V.$$

The above parameters should be used for values of $\Delta E < 3$.

- $\Delta E^*_{94} = \sqrt{\left(\frac{\Delta L^*}{k_L S_L}\right)^2 + \left(\frac{\Delta C^*_{ab}}{k_C S_C}\right)^2 + \left(\frac{\Delta H^*_{ab}}{k_H S_H}\right)^2}$
- ΔL^* - brightness difference,
- $\Delta C{*}_{ab}$ - saturation,
- $\Delta H{*}_{ab}$ - shade for both compared samples,
- S_L, S_C, S_H - compensation coefficients,
- $S_L = 1$, $S_C = 1 + 0.045C^*_{ab}$, $S_H = 1 + 0.015C^*_{ab}$,

where: $C^*_{ab} = \sqrt{[C^*_{ab1} \times C^*_{ab2}]}$ is the geometric mean of the sample value and the pattern, and k_L, k_C, k_H are parametric coefficients to compensate for the interference of external factors in the perception of color difference, with $k_L = k_C = k_H = 1$ for standard lighting conditions.

The ΔE_{94} formula required an additional parameter ΔV, taking into account the change in viewing conditions - a factor k_E: $\Delta V = k_E^{-1} \times \Delta E^*_{94}$, [16].

Despite many efforts and new formulas, there wasn't method of calculating the differences in color that gives the results in numbers related and aligned with the observer's perceived acceptance limits. An additional problem for the difference in color perception are different surface structure coated with color, the observation conditions (lighting, proximity to other colors,size of the plane of measurement) and the reflective characteristics of the surface.

6.6. The ΔE_{2000} formula

In fact, the $CIEL^*a^*b^*$ space is nonuniform. Therefore, the ISO recommends in place of the formula for ΔE_{Lab} use of another formula: ΔE_{2000} (mathematically expanded) especially for determining the color difference in its assessment of industrial.

ΔE_{2000} changes participation of L^* depending on the brightness of the color value range. Measuring pattern has the form:

$$\Delta E_{2000} = \sqrt{(\frac{\Delta L'}{K_L S_L})^2 + (\frac{\Delta C'}{K_C S_C})^2 + (\frac{\Delta H'}{K_H S_H})^2} \tag{3}$$

where:
$\overline{L'} = (L_1 + L_2)/2$,
$\Delta L' = L_2 - L_1$,
$C_1 = \sqrt{a_1^2 + b_1^2}$,
$C_2 = \sqrt{a_2^2 + b_2^2}$,
$\overline{C} = (C_1 + C_2)/2$,
$G = (1 - \sqrt{\frac{\overline{C^7}}{\overline{C^7} + 25^7}})/2$,
$a_1' = a_1(1 + G)$,
$a_2' = a_2(1 + G)$,
$C_1' = \sqrt{a_1'^2 + b_1^2}$,
$C_2' = \sqrt{a_2'^2 + b_2^2}$,
$\overline{C'} = (C_1' + C_2')/2$,
$\Delta C' = C_2' - C_1'$,
$\Delta H' = 2\sqrt{C_1' C_2'} \sin(\Delta h'/2)$,
$S_L = 1 + \frac{0.015(\overline{L}' - 50)^2}{\sqrt{20 + (\overline{L}' - 50)^2}}$,
$S_C = 1 + 0.045\overline{C}'$,
$S_H = 1 + 0.015\overline{C}'T$,
$\Delta\theta = 30 \exp\{-(\frac{\overline{H'} - 275^o}{25})\}$,
$R_C = \sqrt{\frac{\overline{C^7}}{\overline{C^7} + 25^7}})$,
$R_T = -2R_C \sin(2\Delta\theta)$,
$K_L = 1 - default$,
$K_C = 1 - default$,
$K_H = 1 - default$.

7. The ΔE formula in the RGB space

The RGB color space is not an uniform space, so the calculations of distances between colors are inaccurate.

The RGB color model suggests calculation according to the Euclidean distance between points in space representing the RGB colors:

$$\Delta E_{RGB} = \sqrt{\Delta R^2 + \Delta G^2 + \Delta B^2}.$$

However, the human eye has a different sensitivity to changes in the light intensity of the R, G, B components, and changes in the intensity of the individual components produce different sensations. This is taken into account by the following formula with the sensitivity coefficients to compensate for different color components of the eye. It has the form:

$$\Delta E'_{RGB} = \sqrt{3(\Delta R)^2 + 4(\Delta G)^2 + 2(\Delta B)^2}.$$

However, it is still imperfect. Namely, it does not include changes in the sensitivity of the eye caused by changes in the brightness: changes in relatively dark colors are less noticeable for the observer than the same changes in a brighter color. Another example is given by the coefficients of the R and B components from the average value component of R in two compared colors:

$$\Delta E''_{RGB} = \sqrt{(2 + \frac{R_{Sr}}{256})\Delta R^2 + 4\Delta G^2 + (2 + \frac{255 - R_{Sr}}{256})\Delta B^2},$$

where R_{Sr} is the average value of the R components in the two compared colors. Only this last formula gives results suitable for use in graphics software (for 24-bit RGB color model). However, they are not suitable for use either in colorimetry, or printing, or in the paint industry. They are rough, although the calculations are fast.

The same formula can be applied to data from the $CMY(K)$ color model , going first to the RGB space.

8. Examples of ΔE values of close chromas

Below we give the ΔE values in the RGB[11] and $CIELAB$ [12] color spaces, calculated according to the different models (ΔE_{76}, ΔE_{94} and ΔE_{00}) for selected colors such, that ΔE_{76} calculated in the $CIELAB$ space falls into five different ranges, for which:

- $0 < \Delta E_{76} < 1$ - the difference is unnoticeable,
- $1 < \Delta E_{76} < 2$ - the difference is only noticed by an experienced observer,
- $2 < \Delta E_{76} < 3.5$ - the difference is also noticed by an unexperienced observer,
- $3.5 < \Delta E_{76} < 5$ - the difference is clearly noticeable,
- $5 < \Delta E_{76}$ - gives the impression that these are two different colors.

Given a suitable value for ΔE_{RGB}.

For: the $RGB(30, 87, 9)$ and $RGB(31, 88, 10)$ colors, the corresponding ΔE values in the $CIELAB$ and RGB spaces are as follows, Fig. 4a:

$$CIELAB \begin{cases} \Delta E_{76} & = & 0.413838 \\ \Delta E_{CMC(1:1)} & = & 0.482114 \\ \Delta E_{94(G.A.)} & = & 0.404342 \\ \Delta E_{2000(1:1:1)} & = & 0.321896 \end{cases}$$

$$RGB \begin{cases} \Delta E_{RGB} & = & 1.732050. \\ \Delta E'_{RGB} & = & 3. \\ \Delta E''_{RGB} & = & 2.999349. \end{cases}$$

For: the $RGB(255, 0, 0)$ and $RGB(251, 0, 0)$ colors, the corresponding ΔE values in the $CIELAB$ and RGB spaces are as follows, Fig. 4b:

$$CIELAB \begin{cases} \Delta E_{76} & = & 1.493841 \\ \Delta E_{CMC(1:1)} & = & 0.817270 \\ \Delta E_{94(G.A.)} & = & 0.853206 \\ \Delta E_{2000(1:1:1)} & = & 0.835821 \end{cases}$$

[11] ΔE represents the methods described in Section 7

[12] The $RGB- > LAB$ conversion has been made using the D65 white point, 10^o observer angle, and no adaptation

$$RGB \begin{cases} \Delta E_{RGB} & = & 4. \\ \Delta E'_{RGB} & = & 6.928203 \\ \Delta E''_{RGB} & = & 6.914658. \end{cases}$$

Figure 4: Exemplary pairs of two different colors with different individual components RGB

For: the $RGB(255, 25, 137)$ and $RGB(255, 25, 131)$ colors, the corresponding ΔE values in the $CIELAB$ and RGB spaces are as follows, Fig. 4c:

$$CIELAB \begin{cases} \Delta E_{76} & = & 3.401921 \\ \Delta E_{CMC(1:1)} & = & 1.600799 \\ \Delta E_{94(G.A.)} & = & 1.527058 \\ \Delta E_{2000(1:1:1)} & = & 1.322063 \end{cases}$$

$$RGB \begin{cases} \Delta E_{RGB} & = & 6. \\ \Delta E'_{RGB} & = & 8.485281. \\ \Delta E''_{RGB} & = & 8.485251. \end{cases}$$

For: the $RGB(31, 146, 255)$ and $RGB(31, 140, 255)$ colors, the corresponding ΔE values in the $CIELAB$ and RGB spaces are as follows, Fig. 4d:

$$CIELAB \begin{cases} \Delta E_{76} & = & 4.800015 \\ \Delta E_{CMC(1:1)} & = & 2.634044 \\ \Delta E_{94(G.A.)} & = & 2.464112 \\ \Delta E_{2000(1:1:1)} & = & 2.121094 \end{cases}$$

$$RGB \begin{cases} \Delta E_{RGB} & = & 6. \\ \Delta E'_{RGB} & = & 12. \\ \Delta E''_{RGB} & = & 12. \end{cases}$$

For: the $RGB(146, 146, 31)$ and $RGB(131, 131, 31)$ colors, the corresponding ΔE values in the $CIELAB$ and RGB spaces are as follows, Fig. 4e:

$$CIELAB \begin{cases} \Delta E_{76} & = & 7.986917 \\ \Delta E_{CMC(1:1)} & = & 5.481823 \\ \Delta E_{94(G.A.)} & = & 5.918518 \\ \Delta E_{2000(1:1:1)} & = & 5.518652 \end{cases}$$

$$RGB \begin{cases} \Delta E_{RGB} & = & 21.213200. \\ \Delta E'_{RGB} & = & 39.686210. \\ \Delta E''_{RGB} & = & 38.363110. \end{cases}$$

9. Color difference in images

In recent years, many researchers have undertaken work to deduce the color difference formulas for complex images. Here we should mention the S-CIELAB metric ([22], 1996), which is an extension of the ΔE_{ab} formula in the CIELAB space. It has a preprocessing step, a spatial form, including measurements of the sensitivity of color patterns. Other authors (Tremeau et al., [21], 1996) proposed to compare a local color correlation measure based on human visual perception.

9.1. Katoh distance in color

Although the difference in colors should be defined locally, its image perceptibility depends on the context. Katoh et al. in [20] (1996) reworked the difference in CIELAB using psychophysical techniques, including their weight. The formula is:

$$\Delta E_K = \sqrt{\left(\frac{\Delta L^*}{K_L}\right)^2 + \left(\frac{\Delta C^*_{ab}}{K_C}\right)^2 + \left(\frac{\Delta H^*_{ab}}{K_H}\right)^2},$$

where K_L, K_C i K_H are the weight coefficients for chroma and shade clarity.

9.2. Mahalanobis distance in color

There was an urgent need to deduce formulas for the difference in images, as a supplementary formula to the difference in color patches, because these are practical needs. A candidate for such a metric was Mahalanobis distance, taking into account the correlation between the attributes of images [13]

In the article [29] (2001) Imai, Tsumura and Miyake defined differences in color as the Mahalanobis distance with a covariance matrix metric for differences in brightness, color and hue angle between two images. The covariance matrix was obtained as a result of a psychological experiments on a metric for change in brightness, color and hue angle images. These experiments provided some preliminary analysis of potential information, can be separated from the proposed perceptual color difference metric.

This proposal is as follows: the Mahalanobis distance is widely used in recognizing patterns and defining a homogeneous distribution of the impact of each attribute X_1, X_2,$\ldots X_n$. Let us consider correlation between each component:

$$\Delta d = \sqrt{[\Delta X_1 \Delta X_2 \ldots \Delta X_n] \begin{bmatrix} \sigma_{X_1,X_1} & \sigma_{X_1X_2} & \cdots & \sigma_{X_1X_n} \\ \sigma_{X_2X_1} & \sigma_{X_2,X_2} & \cdots & \sigma_{X_2X_n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{X_nX_1} & \sigma_{X_nX_2} & \cdots & \sigma_{X_n,X_n} \end{bmatrix}^{-1} \begin{bmatrix} \Delta X_1 \\ \Delta X_2 \\ \vdots \\ \Delta X_n \end{bmatrix}}$$

where σ_{X_i,X_i} is the variance of attribute X_i, while σ_{X-i,X_J} is the covariance between attributes X_I and X_J.

This variance-covariance matrix can be derived using similar techniques as those used to derive Brown-MacAdam ellipsoids [9]. The authors used the method in which the observer should change

[13] Mahalanobis distance is the distance between two points in an n-dimensional space, which varies the contribution of individual components and uses the correlation between them. Given a set of points forming a class, we can determine the mean vector for $\overline{\mu} = [\mu_1, \mu_2, \ldots, \mu_n]$ and the covariance matrix $\mathbf{C}$ that reflects a certain nature of this class. Studying the affiliation of an unknown random vector **x** to a given class, measured by its similarity to the vector $\overline{\mu}$, takes into account the information about the individual components of variances and correlations between them. The measure of such similarity is the Mahalanobis distance, called the weighted Euclidean distance; the weight matrix is the matrix $\mathbf{C^{-1}}$.

the amount of all three components of the original R, G and B tristimulus variables, and try out fixed excitation. They calculated the variances on the basis of equality:

$$var(R) = \sigma_{R,R} = \frac{1}{n-1}\prod_{i=1}^{n}(R_i - R_0)^2$$
$$var(G) = \sigma_{G,G} = \frac{1}{n-1}\prod_{i=1}^{n}(G_i - G_0)^2$$
$$var(B) = \sigma_{B,B} = \frac{1}{n-1}\prod_{i=1}^{n}(B_i - B_0)^2,$$

where R_i, G_i i B_i are the values of the i-th fold (i=1(1)n), while R_0, G_0 i B_0 re the average values obtained from averaging the color:

$$T_0 = \frac{1}{n}\prod_{i=1}^{n} T_i, \text{ where: T=T(R,G,B).}$$

The covariances can be calculated from the equations:

$$\sigma_{R,G} = cov(R,G) = \sigma_{G,R} = cov(G,R) = \frac{1}{n-1}\prod_{i=1}^{n}(R_i - R_0)(G_i - G_0)$$
$$\sigma_{G,B} = cov(G,B) = \sigma_{B,G} = cov(B,G) = \frac{1}{n-1}\prod_{i=1}^{n}(G_i - G_0)(B_i - B_0)$$
$$\sigma_{R,B} = cov(R,G) = \sigma_{B,R} = cov(B,R) = \frac{1}{n-1}\prod_{i=1}^{n}(B_i - B_0)(R_i - R_0)$$

and are used to construct the variance-covariance matrix:

$$M = \begin{bmatrix} \sigma_{R,R} & \sigma_{R,G} & \sigma_{R,B} \\ \sigma_{G,R} & \sigma_{G,G} & \sigma_{G,B} \\ \sigma_{B,R} & \sigma_{B,G} & \sigma_{B,B} \end{bmatrix}$$

To obtain Brown-MacAdam ellipsoids, elements of matrix M^{-1} are used.

Color attributes, such as brightness metric, color and hue angle, may be linked more closely to human perception than the use of R, G and B. Mahalanobis distance can be employed in the color space using the brightness metric, color and hue angle as shown below:

$$\Delta d = \sqrt{[\Delta L \; \Delta C \; \Delta h] \begin{bmatrix} \sigma_{LL} & \sigma_{LC} & \sigma_{Lh} \\ \sigma_{CL} & \sigma_{CC} & \sigma_{Ch} \\ \sigma_{hL} & \sigma_{hC} & \sigma_{hh} \end{bmatrix}^{-1} \begin{bmatrix} \Delta L \\ \Delta C \\ \Delta h \end{bmatrix}}$$

where: $\sigma_{LL} = W_{LL}$, $\sigma_{CC} = W_{CC}$ i $\sigma_{hh} = W_{hh}$ are the variances of brightness metric, color and hue angle, and ΔL, ΔC and Δh are differences in brightness metric, color and hue angle between two images (for example, original and reproduction). On the other hand, $\sigma_{LC}(\sigma_{CL})$, $\sigma_{LR}(\sigma_{RL})$ i $\sigma_{Ch}(\sigma_{hC})$ are the covariances between the brightness and hue, brightness and hue angle and color and hue angle metrics, respectively. The variance-covariance matrix can be easily derived with the use of 3D perceptibility threshold color difference, as shown below [9, 14, 24]. The values in the last matrix of variances give an indication of how our perception is sensitive to certain images for color and brightness, hue and angle metrics. Taking into account that $W_{LC} = W_{CL}$, $W_{Lh} = W_{hL}$ and $W_{Ch} = W_{hC}$, the last matrix (the expression for Δd) can be written as:

$$\Delta d = \sqrt{W_{LL}\Delta L^2 + W_{CC}\Delta C^2 + W_{hh}\Delta h^2 + 2(W_{LC}\Delta L\Delta C + W_{Lh}\Delta L\Delta h + W_{Ch}\Delta C\Delta h)}$$

The W_{LL} component affects the sensitivity of the perceptual brightness metric, W_{CC} - perceptual color sensitivity, W_{hh} - hue angle sensitivity. $W_{LC}(W_{CL})$ affects the correlation between brightness and color metrics, $W_{Lh}(W_{hL})$ components - on the correlation between brightness and hue angle metric, $W_{Ch}(W_{hC})$ - the correlation between color and hue angle. When the correlation between brightness, color and hue angle metrics does not occur, the distance Δd reduces to the weighted Euclidean distance:

$$\Delta d = \sqrt{W_{LL}\Delta L^2 + W_{CC}\Delta C^2 + W_{hh}\Delta h^2}.$$

In the last dependency we can observe a relationship to the relation with ΔE^*_{94}, which can be derived from the following simplified Mahalanobis model for perceptual difference by substituting:

$$W_{LL} = \left(\frac{1}{k_L s_L}\right)^2,\ W_{CC} = \left(\frac{1}{k_C s_C}\right)^2,\ W_{hh} = \left(\frac{1}{k_h s_h}\right)^2$$

9.3. Luo and Hong spatial difference in a color image

Yet another approach to calculating the perceived color difference was presented by Luo and Hong in [31] (2005). They noted that existing methods of measuring the difference in color for complex images (CIELAB, CMC, CIE94, CIEDE2000 and others) average the difference in each pixel, which is simple to calculate, but does not reflect the difference perceived by the visual system. Therefore, they proposed a new metric better reflecting visual perception. The proposed method of measuring the difference was based on the following observations of the authors, made during psychophysical experiments with comparing color of images:

1. The full difference between images can be calculated as the averaged sum of the differences in color between pixels. Because the CIE color difference formulas are based on colorimetry, andwere derived for homogeneous patches, they should be used for building blocks of a difference image formula. The obvious problem with conventional methods is that any difference in a pixel is weighted equally, although not every pixel is equally important in the image being shown, for example, in the image of a human face, and the eyes attract much more attention than any other part of the picture.
2. Larger areas of the same color should have a higher weight, which results from psychophysical experiments. Experiments aimed at comparing the difference images show that viewers direct sight to the center of certain areas, usually areas with significant size in the image, and give their judgments on the basis of the main difference in the color of these areas. This assumption is consistent with the well-known fact that the human eye tends to be more tolerant to differences in color in small areas of the image.
3. Larger differences in color between pixels should be weighted better. A shortage of current formulas for CIE color differences based on colorimetry is that they are meaningful only for small differences in color. Therefore, they are not suitable for accurate measurement of large differences in color. However for a given image, it is possible that the color of a reproduced pixel or area is quite different from the original, especially when mapped into gamut. The appearance of the

entire image is usually unacceptable when areas show very large difference in color, even when the rest of the image is reproduced well. In the proposed solution, the authors adopted a power of 2 to increase the weights assigned to areas with large differences in color. The results obtained in [27, 23, 18] show that the difference in color equal to 4 ΔE units is acceptable for the difference for comparing images.

4. Visibility of the colour shade is an important distinction within pictorial context. The colour shade of an object is usually dictated by the properties of light absorption or reflection of the material from which the object is made. However, the brightness and the color of the object are actually determined by the lighting and angle of observation.

   In th algorithm, the full range of the CIELAB space hue angle ($0^o \div 360^o$) is compressed by half (the histogram has only 180 different hue angles).

The proposed algorithm for calculating the difference in color images is as follows:

- Convert each pixel from the L^*, a^*, b^* space to the L^*, C^*_{ab}, h_{ab} space.
- Compute the histogram of the image plane h_{ab}, ie, the probability of occurrence of each hue angle, and save the histogram information in the table $hist[hue]$.
- Sort the table $hist[hue]$ in the ascending order, and then divide it into four parts:
  - for the first *n* hue angles in $hist[hue]$, for which $\sum_{i=0}^{n} hist[i] < 25\%$, accept $hist[i] = hist[i]/4$;
  - for the next *m* hue angles in $hist[hue]$, for which $\sum_{i=n+1}^{n+m} hist[i] < 25\%$, accept $hist[i] = hist[i]/2$;
  - for the next *l* hue angles in $hist[hue]$, for which $\sum_{i=n+m+1}^{n+m+l} hist[i] < 25\%$, accept $hist[i] = hist[i]$;
  - for other hue angles in $hist[hue]$ accept $hist[i] = hist[i] * 2 \cdot 25$.
- For each of the existing hue angles, the average difference in the color of all pixels of the same hue angle in the image is computed and saved in $CD[hue]$.
- The overall difference in the color of the entire image is calculated as $CD_{image} = \sum hist[hue] * CD[hue]^2/4$.

Possible changes introduced by the algorithm are arranged in such a way that for most natural images the cumulative probability of all hue angles after modification should be very close to 1, which is the sum of the probabilities of all hue angles in the initial image.

10. Summary

The problem of measuring the color difference has been known for centuries. However, objective measurement of color, became possible only after the introduction of a color space as amathematical model. Yet, not all color spaces are designed for uniform measurement of the color. The most precise measurements are performed in perceptually uniform color spaces such as $CIELAB$. Some inaccuracies in precise measurements, as the Euclidian distance ΔE, were improved by the introduction of newer varieties of ΔE as distance measurement. The most recent of these is $\Delta E2000$.
The need for precise measurement of color is dictated by the development of different industries. The measure most commonly used for general purposes is Euclidian ΔE. Depending on demand more advanced methods of measurement will probably arise!

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